Sequence and Series

Chapter Review

Sequences - What Are They?

$1, 3, 5, 7, 9, \ldots$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

$1, 3, 5, 7, 9, \ldots$

"Terms"

$1, 3, 5, 7, 9, \ldots, \underline{\quad}, \ldots$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_n$

A sequence is a function whose domain is the set of positive integers. The function values

$$a_1, a_2, a_3, a_4, \ldots a_n, \ldots$$

are the terms of the sequence. If the domain consists of the first n positive integers only, then the sequence is finite.

Factorials - What Are They?

$$\boxed{n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots (n-1) \cdot n} \qquad \boxed{0! = 1}$$

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

$$\frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 8 \cdot 7 \cdot 6$$

🚩 Series - What Are They?

A series is the sum of the first n terms of a sequence.

$$\text{Sum} = a_1 + a_2 + a_3 + \cdots\, a_n$$

$$\text{Sum} = 1 + 2 + 3 + 4 + 5$$

$$\underbrace{a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5}_{\substack{n=5 \\ 5 \text{ terms}}}$$

Summation Notation - "Sigma" Σ

An easy way to express a series

$$\boxed{\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots a_n}$$

$$\sum_{i=1}^{n} a_i$$

$n \leftarrow$ upper limit of summation

$i = 1 \leftarrow$ lower limit of summation

i: index of summation

" the sum from $i=1$ to n of a_i "

$$\text{Sum} = \underbrace{\underset{a_1}{1} + \underset{a_2}{2} + \underset{a_3}{3} + \underset{a_4}{4} + \underset{a_5}{5}}_{\substack{n=5 \\ 5 \text{ terms}}}$$

can write this way

$$\sum_{i=1}^{5} a_i$$

Summation Notation - Examples

$$\sum_{i=1}^{4} 2i = 2(1) + 2(2) + 2(3) + 2(4)$$

$$= 2 + 4 + 6 + 8$$

$$= 20$$

$$\sum_{k=2}^{6} (1+k) = (1+(2)) + (1+(3)) + (1+(4))$$

$$+ (1+(5)) + (1+(6))$$

$$= 3 + 4 + 5 + 6 + 7$$

$$= 25$$

$$\sum_{j=0}^{3} j! - 3 = [(0)! - 3] + [(1)! - 3] +$$

$$[(2)! - 3] + [(3)! - 3]$$

$$= (0! - 3) + (1! - 3) + (2! - 3) + (3! - 3)$$

$$= (1-3) + (1-3) + (2-3) + (6-3)$$

$$= -2 + -2 + -1 + 3$$

$$= -5 + 3$$

$$= -2$$

Formulas For Special Series

$$\boxed{\sum_{i=1}^{n} 1 = n}$$

$$\boxed{\sum_{i=1}^{n} i = \frac{1}{2}(n)(n+1)}$$

$$\boxed{\sum_{i=1}^{n} i^2 = \frac{1}{6}(n)(n+1)(2n+1)}$$

$$\sum_{i=1}^{16} i^2 = 1^2 + 2^2 + 3^2 + \cdots 16^2$$

$$\sum_{i=1}^{16} i^2 = \frac{1}{6}(16)(16+1)(2(16)+1)$$

$$= \frac{1}{6}(16)(17)(33) = 1496$$

Review

1. A sequence is a collection of numbers with a pattern. However, in mathematics the definition of a sequnce is more detailed.

2. A series is the sum of a sequence.

3. A sequence and series can either be finite or infinite.

3. A factorial is an operation that is often found in very important squences in mathematics. Examples: 5! = 5*4*3*2*1 and 0! = 1.

4. Summation Notation - "Sigma" is a convenient notation used for the sum of terms of a sequence.

5. Formulas for special series are more efficient methods to finding the sum of many terms.

Arithmetic Sequences and Series

Arithmetic Sequences and Series - What Are They?

2, 5, 8, 11, 14

the sequence $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$ is arithmetic if there is a number d such that

$$a_2 - a_1 = d, \quad a_3 - a_2 = d, \quad a_4 - a_3 = d$$

and so on. The number d is the common difference of the arithmetic sequence.

2, 5, 8, 11, 14

$5 - 2 = 3 \qquad 8 - 5 = 3 \qquad 11 - 8 = 3$

$$\underbrace{2 + 5 + 8 + 11 + 14}_{\text{Arithmetic Series}}$$

Finding The nth Term of An Arithmetic Sequence

$$7,\ 11,\ 15,\ 19,\ \ldots,\ 4n+3$$

$$a_1 \quad a_2 \quad a_3 \quad a_4 \quad \ldots \quad a_n$$

$a_n = 4n+3$

$a_4 = 4(4)+3$

$a_4 = 16+3 = 19$

Every arithmetic sequence has an n^{th} term in the form of

$$\boxed{a_n = a_1 + (n-1)d}$$

$$7,\ 11,\ 15,\ 19,\ \ldots,\ a_n$$

how do we find a_{100}?

Well, we need the $a_n =$ formula

$a_n = a_1 + (n-1)d$

$a_n = 7 + (n-1)4$

$a_n = 7 + 4n - 4$

$a_n = 4n + 3$

$a_{100} = 4(100) + 3$

$a_{100} = 403$

Examples: Finding The nth Term of An Arithmetic Sequence

Find a formula for the nth term of the arithmetic sequence whose first term is 2 and whose common difference is 3.

$a_n = a_1 + (n-1)d$ $\qquad$ $2, 5, 8, 11, \ldots, 3n-1, \ldots$

$a_n = 2 + (n-1)3$

$a_n = 2 + 3n - 3$

$a_n = 3n - 1$

The fourth term of an arithmetic sequence is 16. The 13th term is 43. Write a formula for the nth term.

$a_4 = 16 \longrightarrow a_1 = 16$ $\leftarrow$ you need a_1 to use the formula

$a_{13} = 43 \longrightarrow a_{10} = 43$

$a_n = a_1 + (n-1)d$

$a_{10} = 43$

we have a_{10}

$n = 10$

have this

need to solve for d

$a_4 = 16 \longrightarrow a_1 = 16$

$a_{13} = 43 \longrightarrow a_{10} = 43$

$a_n = a_1 + (n-1)d$

$43 = 16 + (10-1)d$

$43 = 16 + 9d$

$27 = 9d$

$d = 3$ $\longleftarrow$ Now that we have d we can get the real a_1

$a_4 = 16 \longrightarrow a_1 =$

3 terms away

$d = 3$ $\quad a_1, a_2, a_3, a_4$

$a_1 = 7$ $\quad 7 \quad 10 \quad 13 \quad 16$

$a_1 = a_4 - 3(d)$

$a_1 = 16 - 3(3) = 7$

$a_1 = 7$

$a_1 = 7 \longrightarrow$
$d = 3 \longrightarrow$ $\boxed{a_n = a_1 + (n-1)d}$

$a_n = 7 + (n-1)3$

$a_n = 7 + 3n - 3$

$a_n = 3n + 4$

How to Find The Sum of An Arithmetic Series

The sum of an arithmetic series with n terms is

$$\sum_{i=1}^{n} a_i = n\left(\frac{a_1 + a_n}{2}\right)$$

$$S = \frac{1}{2}n\left[2a_1 + (n-1)d\right]$$

$$\underbrace{1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21}_{\substack{11 \text{ terms} \\ (n = 11)}} = ?$$

$a_1 = 1$ $d = 2$ $a_{11} = 21$

$$\boxed{\sum_{i=1}^{n} a_i = n\left(\frac{a_1 + a_n}{2}\right)}$$

$$\sum_{i=1}^{11} a_i = 11\left(\frac{1 + 21}{2}\right)$$

$$= 11\left(\frac{22}{2}\right)$$

$$\text{Sum} = 11(11) = 121$$

$$\boxed{S = \frac{1}{2}n\left[2a_1 + (n-1)d\right]}$$

$$= \frac{1}{2}(11)\left[2(1) + (11-1)2\right]$$

$$= \frac{1}{2}(11)\left[2 + 20\right]$$

$$= \frac{1}{2}(11)(22)$$

$$= (11)(11) = 121$$

Review

1. An arithmetic sequence is a sequence where each term is separated by a common difference.

2. Every arithmetic sequence has an nth term in the form of: $a_n = a_1 + (n-1)d$

3. We can find the nth term of an arithmetic sequence by using the general formula (above).

4. We can find the sum of an arithmetic series by using the formulas:

$$\sum_{i=1}^{n} a_i = n\left(\frac{a_1 + a_n}{2}\right)$$

$$S = \frac{1}{2}n\left[2a_1 + (n-1)d\right]$$

Geometric Sequences and Series

Geometric Sequences and Series - What Are They?

$2, 4, 8, 16, 32, \ldots$

the sequence $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$ is geometric if there is a non-zero number r such that

$$\frac{a_2}{a_1} = r, \quad \frac{a_3}{a_2} = r, \quad \frac{a_4}{a_3} = r, \ldots$$

the number r is the common ratio of the geometric sequence.

$2, 4, 8, 16, 32, \ldots$

$$\frac{4}{2} = 2, \quad \frac{8}{4} = 2, \quad \frac{16}{8} = 2, \ldots$$

$r = 2$

Finding The nth Term of A Geometric Sequence

$$2, 4, 8, 16, 32, \ldots, ?, \ldots$$

$$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad \ldots \quad a_n$$

Every geometric sequence has an n^{th} term in the form of

$$\boxed{a_n = a_1 r^{n-1}}$$

$$2, 4, 8, 16, \ldots, a_n$$

how do we find a_{15}?

$r = 2 = \frac{4}{2}$

Well, we need the $a_n =$ formula

$$a_n = a_1 r^{n-1}$$
$$a_n = 2(2)^{n-1}$$

$$a_{15} = 2(2)^{15-1}$$
$$a_{15} = 2(2)^{14}$$
$$a_{15} = 2(16384)$$
$$a_{15} = 32768$$

Examples: Finding The nth Term of An Geometric Sequence

Find a formula for the nth term of the geometric sequence whose first three terms are 5, 15, 45. Then find the 12th term.

$$a_n = a_1 r^{n-1}$$

$$r = \frac{15}{5} = \frac{45}{15} = 3$$

$$a_n = 5(3)^{n-1}$$

$$a_n = 5(3)^{n-1}$$

$$a_{12} = 5(3)^{12-1}$$

$$a_{12} = 5(3)^{11}$$

$$a_{12} = 5(177147)$$

$$= 885,735$$

The fourth term of a geometric sequence is 125. The 10th term is 125/64. Find the first term.

$$a_4 = 125 \longrightarrow a_1 = 125$$ ← you need a_1 to use the formula

$$a_{10} = \frac{125}{64} \longrightarrow a_7 = \frac{125}{64}$$

$$a_n = a_1 r^{n-1}$$

$a_7 = \frac{125}{64}$ we have

have this $a_1 = 125$

we need to solve for r

$n = 7$ we have

$a_4 = 125 \longrightarrow a_1 = 125$

$a_{10} = \frac{125}{64} \longrightarrow a_7 = \frac{125}{64}$

$a_n = a_1 r^{n-1}$

$a_1 = 125$

$a_7 = 125 r^{(7-1)}$

$a_7 = 125 r^6$

$a_7 = \frac{125}{64}$

$\frac{125}{64} = 125 r^6$

$\frac{1}{64} = r^6$

$\sqrt[6]{r^6} = \sqrt[6]{\frac{1}{64}}$

$\sqrt[6]{\frac{1}{64}} = \frac{\sqrt[6]{1}}{\sqrt[6]{64}}$

$(r^6)^{1/6} = \left(\frac{1}{64}\right)^{1/6}$

$\sqrt[6]{64} = (64)^{1/6} = 2$

$r = \frac{1}{2}$

🚩 64 ^ (1/6)

$$\left.\begin{array}{l} a_n = a_1 r^{n-1} \\ a_4 = 125 \\ r = \frac{1}{2} \end{array}\right\} \longrightarrow \begin{array}{l} a_4 = a_1 \left(\frac{1}{2}\right)^{4-1} \\ a_4 = a_1 \left(\frac{1}{2}\right)^3 \\ 125 = a_1 \left(\frac{1}{8}\right) \\ a_1 = 1000 \end{array}$$

How to Find The Sum of A Finite Geometric Series

the sum of a <u>FINITE</u> geometric series

$$a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1}$$

with a common ratio $r \neq 1$ is

$$\sum_{j=1}^{n} a_1 r^{i-1} = a_1 \left(\frac{1 - r^n}{1 - r} \right)$$

$$\sum_{i=1}^{12} 4(0.3)^i = 4(0.3) + 4(0.3)^2 + \cdots 4(0.3)^{12}$$

$$\sum_{j=1}^{n} a_1 r^{i-1} = a_1 \left(\frac{1 - r^n}{1 - r} \right)$$

$n = 12$

$4(0.3)$

0.3

$$\sum_{j=1}^{12} 4(0.3)^i = 1.2 \left[\frac{1 - (.3)^{12}}{1 - (.3)} \right]$$

$$\approx 1.714$$

Review

1. A geometric sequence is a sequence where each term is seperated by a common ratio.

2. Every geometric sequence has an nth term in the form of:

$$a_n = a_1 r^{n-1}$$

3. We can find the nth term of an geometric sequence by using the general formula (above).

4. We can find the sum of a finite geometric series by using the formula:

$$\sum_{i=1}^{n} a_1 r^{i-1} = a_1\left(\frac{1-r^n}{1-r}\right)$$

Infinite Geometric Series

Infinite Geometric Series - What Are They?

$$a_1, \; a_2, \; a_3, \; \ldots, \; \ldots$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 1 & 1/2 & 1/4 \end{array}$$

$$a_1 = 1$$

$$r = 1/2$$

$$a_1 + a_2 + a_3 + \cdots, \; a_\infty$$

$$\underbrace{1 + \frac{1}{2} + \frac{1}{4} + \cdots, \; \frac{1}{10{,}000}, \; + \cdots \; \frac{1}{\infty}}_{\text{Sum} = 2}$$

🚩 How to Find The Sum of An Infinite Geometric Series

Sum of an infinite geometric series

if $|r|<1$, then the infinite geometric series $a_1 + a_1 r + a_1 r^2 + \dots a_1 r^{n-1} + .$ has the sum

$$\sum_{n=1}^{\infty} a_1 r^{n-1} = \frac{a_1}{1-r}$$

if $|r| \geq 1$, then the series has no sum.

$$1 + \frac{1}{2} + \frac{1}{4} + \dots, \frac{1}{10{,}000}, + \dots \frac{1}{\infty}$$

$$\sum_{n=1}^{\infty} a_1 r^{n-1} = \frac{a_1}{1-r} \quad \begin{matrix} \leftarrow 1 \\ \leftarrow r = 1/2 \end{matrix}$$

$|r|<1$

$|1/2|<1$

$$\sum_{n=1}^{\infty} 1\left(\frac{1}{2}\right)^{n-1} = \frac{1}{1-1/2} = \frac{1}{\frac{1}{2}} = 2$$

Review

1. An infinite geometric series is a geometric series with infinite terms that has a finite sum.

2. We can find the sum of an infinite geometric series by using the formula:

$$\sum_{n=1}^{\infty} a_1 r^{n-1} = \frac{a_1}{1-r}, \quad |r|<1$$

The Binomial Theorem

Review the Patterns of a Binomal Expansion

$$(x+y)^0 = 1$$
$$(x+y)^1 = x + y$$
$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$
$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

$$(x+y)^0 = 1$$
$$(x+y)^1 = 1 + 1$$
$$(x+y)^2 = 1 + 2 + 1$$
$$(x+y)^3 = 1 + 3 + 3 + 1$$
$$(x+y)^4 = 1 + 4 + 6 + 4 + 1$$
$$(x+y)^5 = 1 + 5 + 10 + 10 + 5 + 1$$

Pascal's Triangle

power of the binomial

	1	row 0
$(x+y)^1$	1 1	row 1
$(x+y)^2$	1 2 1	row 2
	1 3 3 1	row 3
	1 4 6 4 1	row 4
	1 5 10 10 5 1	row 5
	1 6 15 20 15 6 1	row 6
	1 7 21 35 35 21 7 1	row 7

1	7	21	35	35	21	7	1
Term 0	Term 1	Term 2	Term 3	Term 4	Term 5	Term 6	Term 7

🚩 Binomial Coefficients - What They Are and How To Find Them

power of the binomial

					1				row 0
$(x+y)^1$					1 1				row 1
$(x+y)^2$					1 2 1				row 2
					1 3 3 1				row 3
					1 4 6 4 1				row 4
					1 5 10 10 5 1				row 5
					1 6 15 20 15 6 1				row 6
					1 7 21 35 35 21 7 1				row 7

1	7	21	35	35	21	7	1
T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7

$$\boxed{\binom{n}{k} = \frac{n!}{k!(n-k)!}}$$

Quick Review n!

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$0! = 1$

"Given n choose k"

$_nC_k$ $\qquad C(n,k)$ $\qquad \binom{n}{k}$ ← row, ← term

							1									row 0
						1		1								row 1
					1		2		1							row 2
				1		3		3		1						row 3
$\binom{6}{4}$			1		4		6		4		1					row 4
		1		5		10		10		5		1				row 5
	1		6		15		20		15		6		1			row 6
1		7		21		35		35		21		7		1		row 7

↑ Term 0 ↑ Term 1 ↑ Term 2 ↑ Term 3 ↑ Term 4 ↑ Term 5 ↑ Term 6 ↑ Term 7

Row → n, Term → k

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \qquad \binom{6}{4} = 15$$

$$\binom{6}{4} = \frac{6!}{4!(6-4)!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{4\cdot3\cdot2\cdot1\,(2\cdot1)}$$

$$= \frac{6\cdot5\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}\,(2\cdot1)}$$

$$= \frac{6\cdot5}{2\cdot1} = \frac{30}{2} = 15$$

1 ← row 0

$1 \quad 1$ ← row 1

$1 \quad 2 \quad 1$ ← row 2

$1 \quad 3 \quad 3 \quad 1$ ← row 3

$1 \quad 4 \quad 6 \quad 4 \quad 1$ ← row 4

$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$ ← row 5

$1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1$ ← row 6

$1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$ ← row 7

$\binom{8}{0} \; \binom{8}{1} \; \binom{8}{2} \; \binom{8}{3} \; \binom{8}{4} \; \binom{8}{5} \; \binom{8}{6} \; \binom{8}{7} \; \binom{8}{8}$ ← row 8

Row → n

term → k

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \qquad \binom{8}{3} = \frac{8!}{3!(8-3)!}$$

$$= \frac{8!}{3!(5!)} = \frac{8\cdot7\cdot6\cdot\cancel{5!}}{3!\cdot\cancel{5!}} = \frac{8\cdot7\cdot6}{3\cdot2} =$$

$$= \frac{8\cdot7\cdot\cancel{6}}{\cancel{6}} = 56$$

calculator

$\boxed{{}_nC_r}$

$${}_8C_3 = \binom{8}{3} = 56$$

Short Cut To Keep In Mind...

$$\binom{6}{4} = 15$$

$$\binom{6}{4} = \frac{\overbrace{6 \cdot 5 \cdot 4 \cdot 3}^{\text{4 factors}}}{\underbrace{4 \cdot 3 \cdot 2 \cdot 1}_{\text{4 factors}}} = \frac{6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3}}{\cancel{4} \cdot \cancel{3} \cdot 2 \cdot 1} = \frac{30}{2} = 15$$

4 factors / factorials

$$\binom{8}{3} = 56$$

$$\binom{8}{3} = \frac{\overbrace{8 \cdot 7 \cdot 6}^{\text{3 factors}}}{\underbrace{3 \cdot 2 \cdot 1}} = \frac{8 \cdot 7 \cdot \cancel{6}}{\cancel{3} \cdot \cancel{2} \cdot 1} = 56$$

3 factors / factorials

The Binomial Theorem

if n is a positive integer, then for any real numbers a and b

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

where $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Row $\rightarrow$ n
Term $\rightarrow$ r $\binom{n}{r}$

$$(x+y)^7 = \binom{7}{0}x^7y^0 + \binom{7}{1}x^6y^1 + \binom{7}{2}x^5y^2$$

$$+ \binom{7}{3}x^4y^3 + \binom{7}{4}x^3y^4 + \binom{7}{5}x^2y^5$$

$$+ \binom{7}{6}x^1y^6 + \binom{7}{7}x^0y^7$$

$$= 1x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4$$

$$+ 21x^2y^5 + 7xy^6 + 1y^7$$

$$(2x-y)^3$$

$$\begin{array}{ccccccccl}
 & & & 1 & & & & & \text{row } 0 \\
 & & 1 & & 1 & & & & \text{row } 1 \\
 & 1 & & 2 & & 1 & & & \text{row } 2 \\
1 & & 3 & & 3 & & 1 & & \text{row } 3
\end{array}$$

$$(2x-y)^3 = 1(2x)^3(-y)^0 + 3(2x)^2(-y)^1$$

$$+\ 3(2x)^1(-y)^2 + 1(2x)^0(-y)^3$$

$$= (2x)^3 + 3(4x^2)(-y) + 3(2x)(y^2) + (-y)^3$$

$$= 8x^3 - 12x^2y + 6xy^2 - y^3$$

How to Find a Single Term of a Binomial Expansion

the r^{th} term of the binomial expansion $(x+y)^n$ where $n \geq r-1$ is

$$\binom{n}{r-1} x^{n-(r-1)} y^{r-1}$$

Find the 4^{th} term of the expansion $(a+2b)^{10}$

$$(a+2b)^{10} = T1 + T2 + T3 + T4 + T5 + T6 + T7 + T8 + T9 + T10 + T11$$

?

the r^{th} term of the binomial expansion $(x+y)^n$ where $n \geq r-1$ is

$$\binom{n}{r-1} x^{n-(r-1)} y^{r-1}$$

$r = 4$ (4^{th} term)

$n = 10$

$n \geq r-1$

$10 \geq (4-1)$ ✓

$$\binom{n}{r-1} = \binom{10}{3}$$

the r^{th} term of the binomial expansion $(x+y)^n$ where $n \geq r-1$ is

$$\binom{n}{r-1} x^{n-(r-1)} y^{r-1}$$

$r = 4$ (4^{th} term)

$n = 10$

$n \geq r-1$

$10 \geq (4-1)$ ✓

$$\binom{n}{r-1} = \binom{10}{3}$$

$$\begin{array}{c} (x+y)^n \\ (a+\underbrace{2b}_{})^{10} \\ \uparrow \quad \uparrow \\ x \quad y \end{array} \implies \binom{10}{3} a^{10-(4-1)} (2b)^{(4-1)}$$

$$= 120\, a^{10-(3)} (2b)^3$$

$$= 120 a^7 (8b^3)$$

$$= 960 a^7 b^3$$

Review

1. The pattern of binomial expansions form Pascal's Triangle.

2. Binomial Coefficients (which are the numbers in Pascal's Triangle) can be found by using the formula:

$$\begin{array}{r} \text{Row} \longrightarrow \\ \text{term} \longrightarrow \end{array} \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

3. The Binomial Theorem is used to expand any binomial.

4. We can find use the Binomial Theorem and binomial coefficients to find a single term of a binomial expansion.